The Meaning of System Strength, Nonlinear Oscillations and Everything

(42!)

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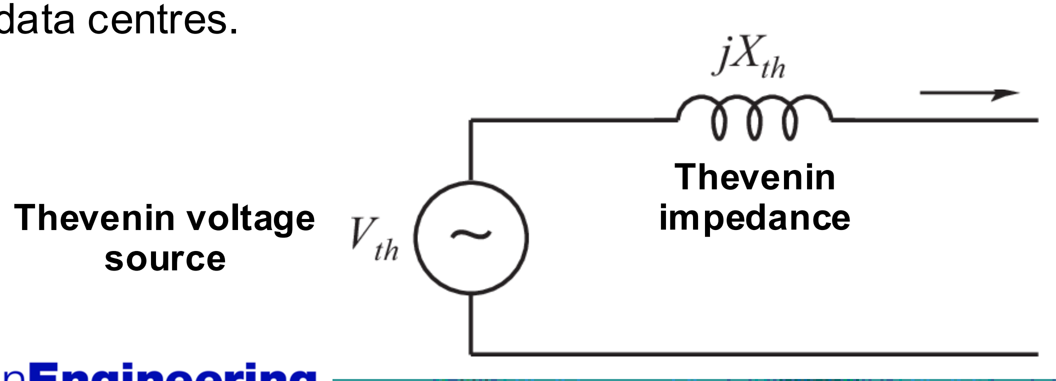
Outline

- System strength
- Hybrid dynamics
- Sustained (nonlinear) oscillations
- (Okay, not quite everything!)

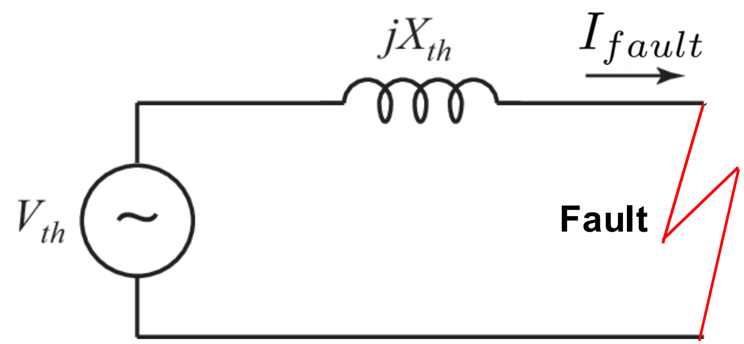
System strength

- The concept of power system strength has been around forever.
  - It is meant to capture the **sensitivity of voltage to disturbances**.
  - It has nothing to do with system inertia (though devices such as synchronous generators provide both.)
- It is traditionally related to the concept of fault level.
  - This is a convenient, but not necessary, relationship.
- Fault level:
  - How much current will flow if the power system experiences a short-circuit?
- Consider the "Thevenin equivalent" of a power system.
  - This "equivalent" can be computed for any point in the power system.
  - Assumes a linear network (but power systems are not linear).
  - This linearity assumption is particularly bad for "constant power" loads such as data centres.



Fault level

- Consider a short-circuit that is located at some point in the power system.
- The current that flows in response to that fault can be determined (approximately) from the Thevenin equivalent.



Ohm's law gives:

$$I_{fault} = \frac{V_{th}}{jX_{th}}$$

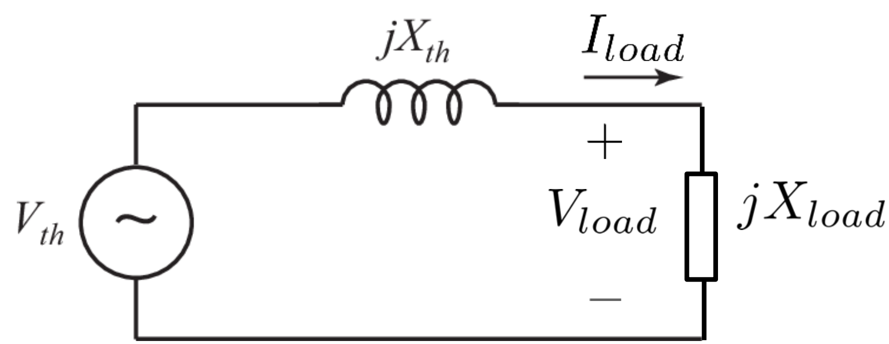
Fault level:

$$S_{fault} \approx 1/X_{th}$$

- If X_{th} is small (S_{fault} is large) then I_{fault} is large.
  - High fault current facilitates fast acting protection, but circuit breakers must be appropriately rated.
- If X_{th} is large (S_{fault} is small), then fault current is small, and protection may not operate reliably.
  - One of the most problematic issues facing renewable-based power systems.

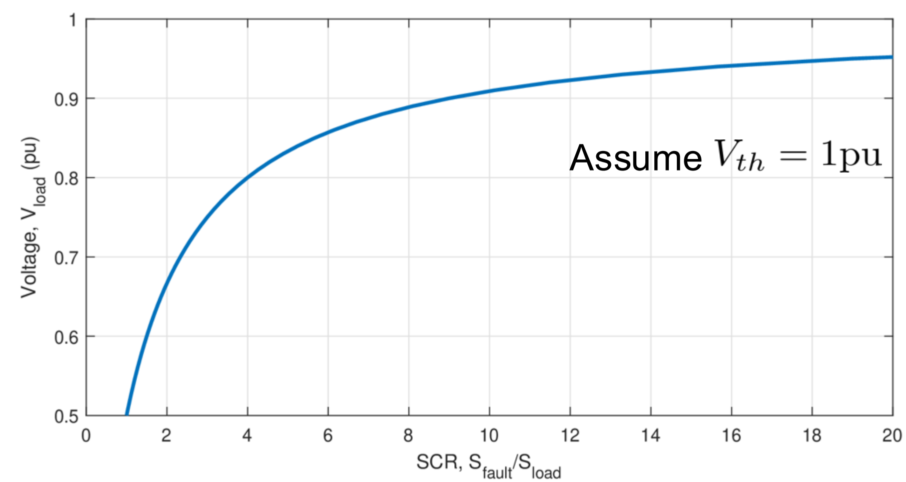
Short circuit ratio (SCR)

- Again consider the Thevenin equivalent, now supplying a load given by the impedance jX_{load} .
- The power drawn by the load is (approximately), $S_{load} \approx 1/X_{load}$.



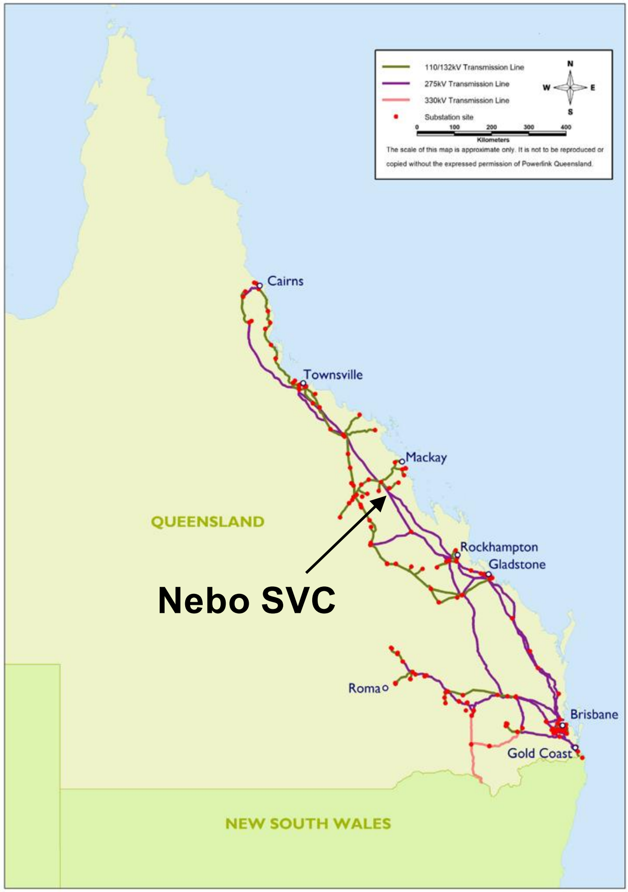
- Simple manipulation of the voltage divider equation gives:

$$SCR = \frac{S_{fault}}{S_{load}} = \frac{V_{load}}{V_{th} - V_{load}}$$



- This is the supply capability relative to load.
- SCR is the traditional measure of system strength.

Example: Nebo SVC



- Low SCR can be managed.
- Planning studies in the early 1980s identified the need for voltage support on the 275kV transmission network between Central and North Queensland.
  - Northern generators were vulnerable to instability.
  - Solution: Nebo static var compensator (SVC), with a dynamic range of 340MVAr (80MVAr inductive to 260MVAr capacitive).
- The Nebo fault level was ~1100MVA, giving:

$$SCR = \frac{1100}{340} = 3.2$$

- The SVC had the potential to drive the voltage to ~1.3pu if the controller misbehaved.
  - It has provided reliable voltage support since commissioning in August 1987.

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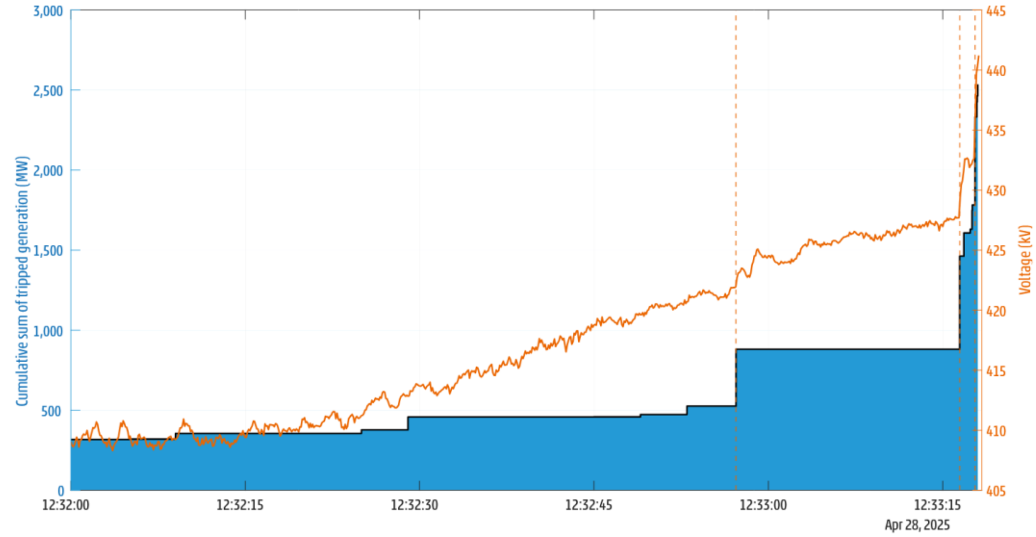
Example: Iberian blackout

- High SCR can be disastrous.

- The Iberian power system experienced oscillations.
- Operators switched numerous transmission lines into service.
  - The reason: **mitigate the oscillations by strengthening the system**.



- Transmission lines were lightly loaded, so they produced a substantial amount of reactive power.
- This excessive reactive power drove voltages high across the transmission network.
- Generators absorbed reactive power but encountered limits and subsequently tripped, causing voltages to increase further.

System strength

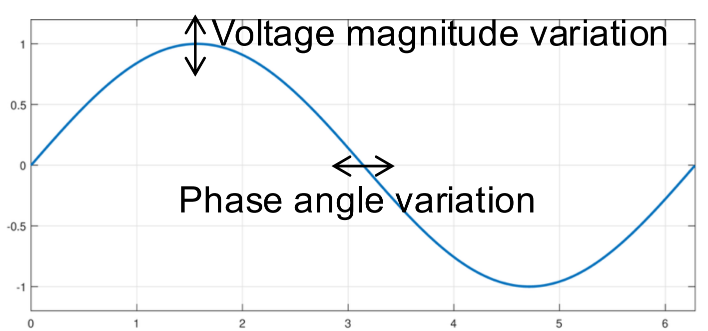
- Short circuit ratio (SCR) is an approximate, back of the envelope, type of calculation.
- The real world is somewhat more complicated than the assumptions underpinning SCR.
- But what is a good measure?

- AEMC: "a characteristic of an electrical power system that relates to the size of the change in voltage following a fault or disturbance".
- AEMO: "the ability of the power system to maintain and control the voltage waveform", "is proportional to the fault level at that location, inversely proportional to" generation/load.
- Powerlink: "a measure of the ability of the power system to remain stable by maintaining the voltage waveform".

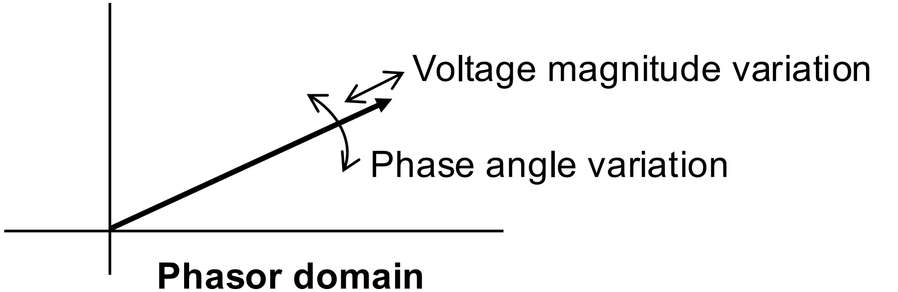
- AEMC, Managing power system fault levels, Rule Determination, 19 September 2017.
- AEMO, System strength in the NEM explained, March 2020.
- Powerlink, 2024 Transmission Annual Planning Report, 2024.

Voltage sensitivity

- Any reasonable concept of system strength must provide a measure of sensitivity of voltage to disturbances.
  - Large perturbations challenge the tracking ability of phase-locked loops.
- A “perfect” power system is one where voltages are insensitive to changes in load/generation.
  - Then every device connected to the network is decoupled from every other device.
- Traditionally, voltage sensitivity has only considered voltage magnitudes.
  - But keep in mind, voltages are actually sinusoidal waveforms.
  - Sensitivity of both peak value and phase angle is necessary.



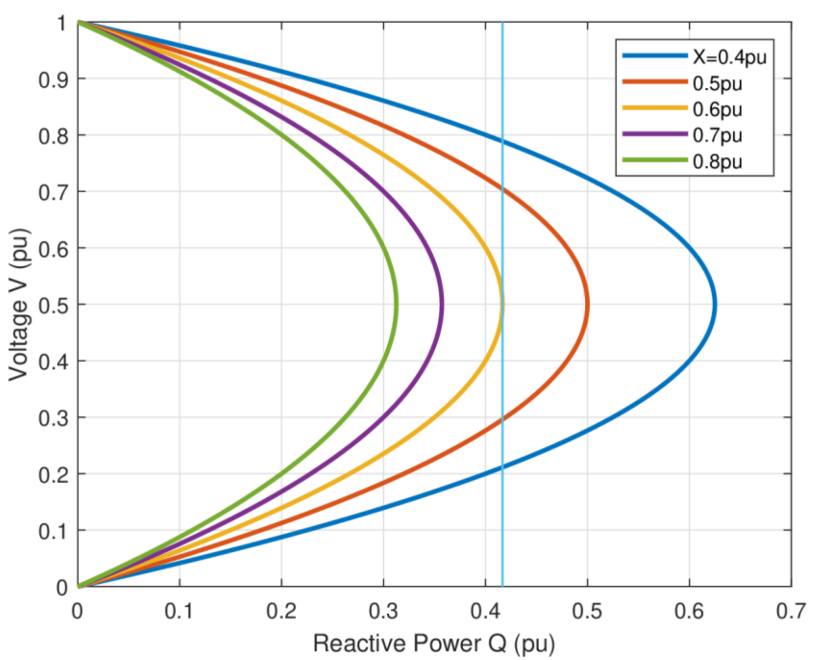
Time domain

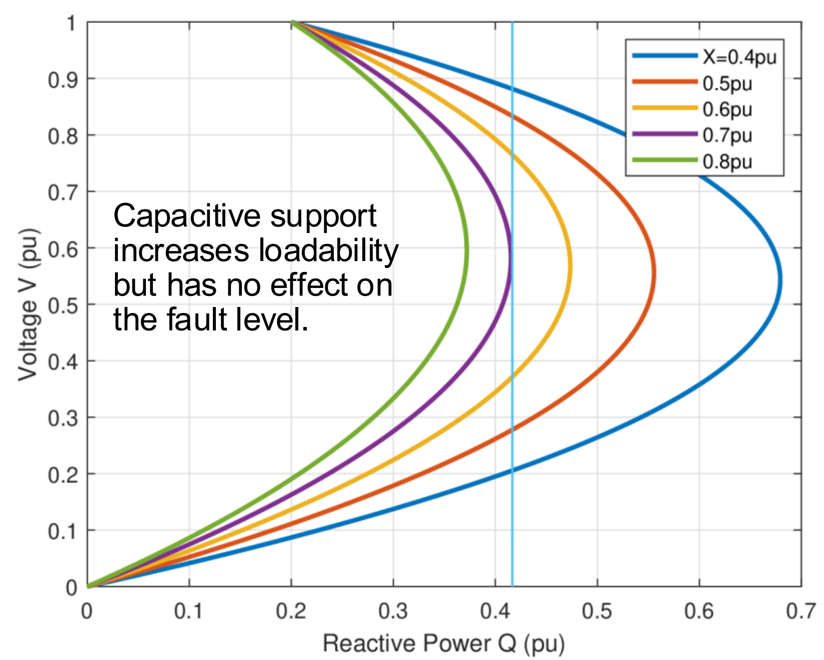


Phasor domain

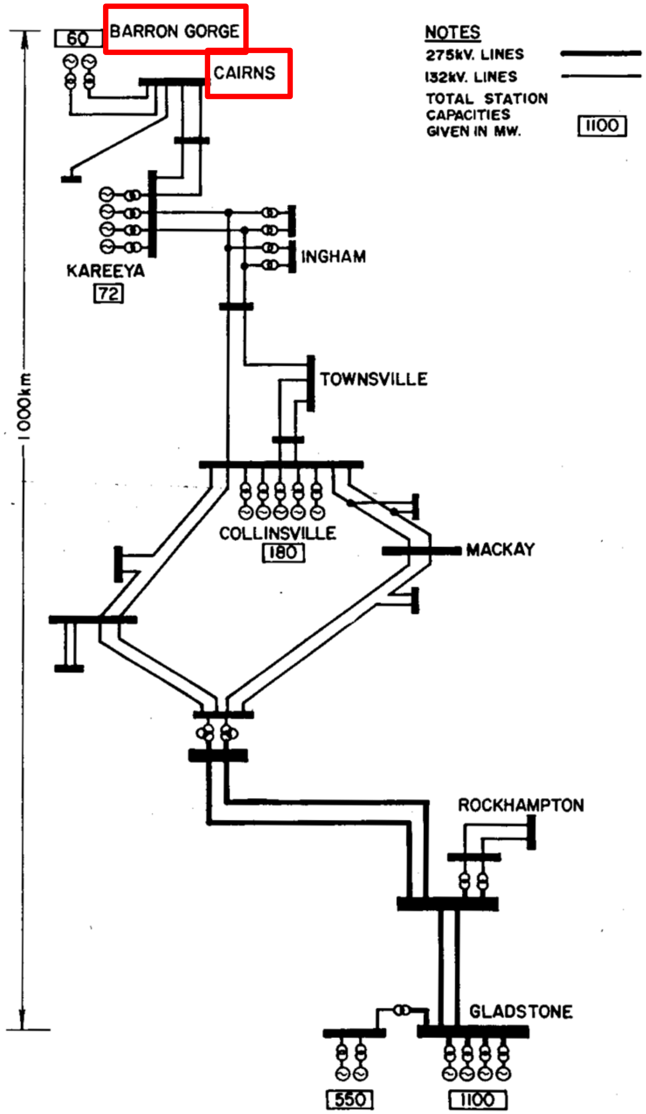
Moving beyond SCR

- A much more complete (and accurate) understanding of system strength and network loadability is available from the nonlinear power flow equations.
- Example: a 2-bus network.
  - The curves show network supply capability for increasing values of network impedance.
  - The vertical line indicates the (reactive power) load.
  - The intersection between a curve and the line provides the operating point.

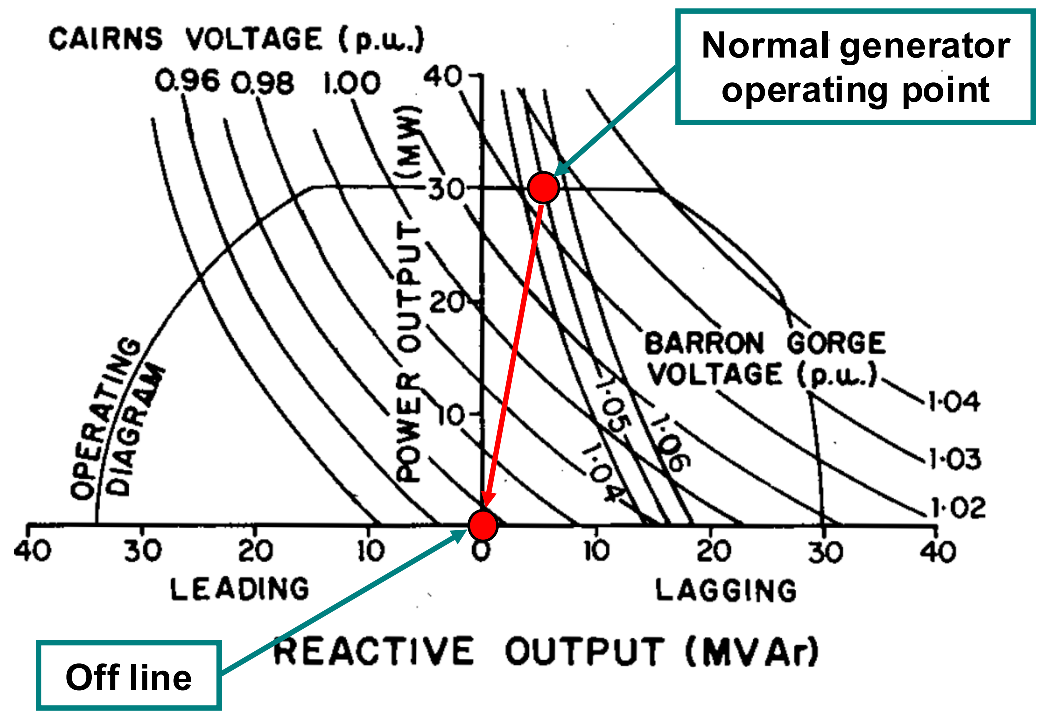




Example: Far North Queensland

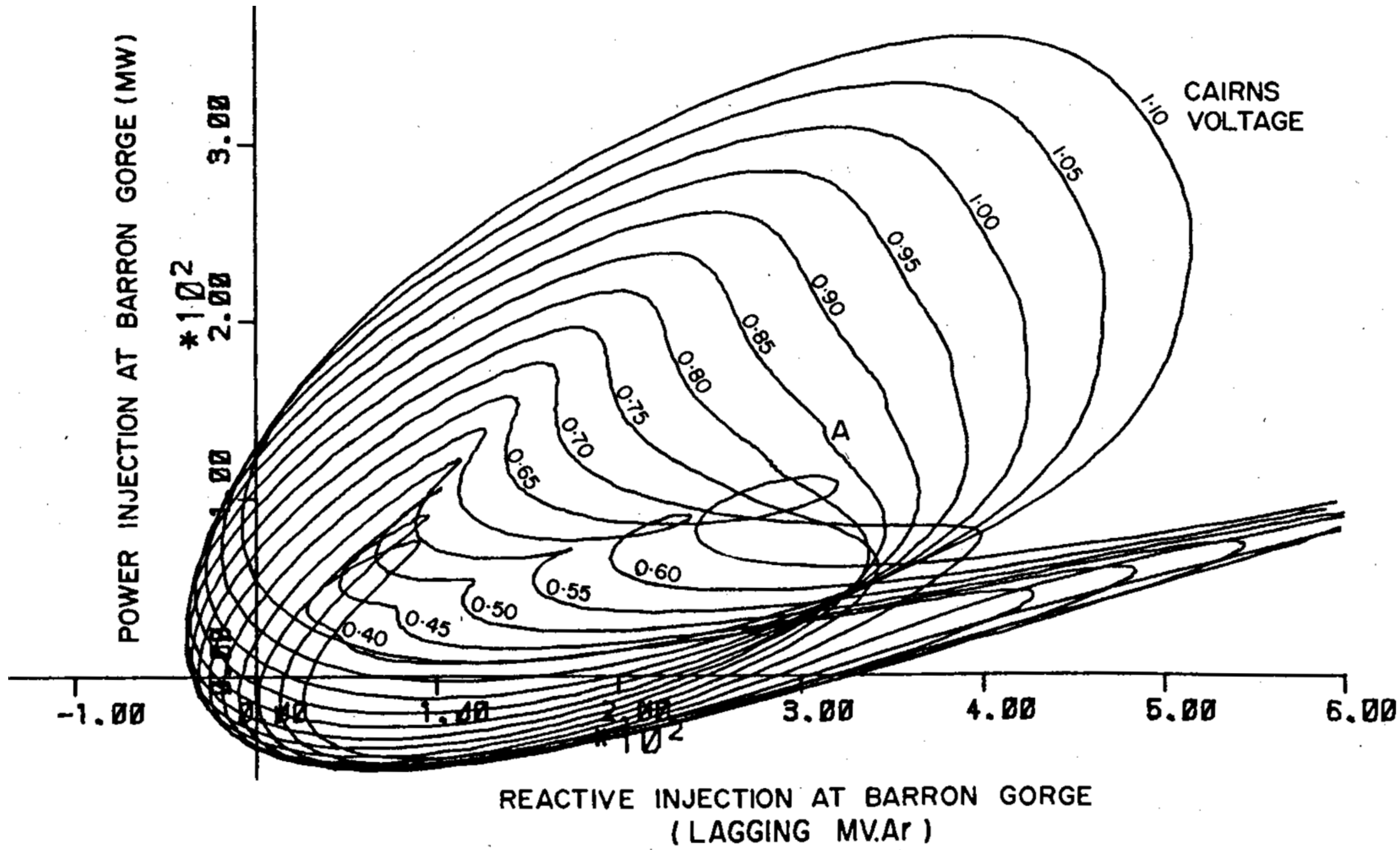


- Vertical and horizontal axes show active and reactive power, respectively, generated a Barron Gorge unit.
- Contours show voltage magnitude at Cairns.
- The red arrow shows voltage magnitude at Cairns drops from 1.024pu to 0.967pu when a Barron Gorge unit trips.



Far North Queensland (cont.)

- A more complete picture of the Cairns – Barron Gorge interactions:



G.B. Price, "A Generalized Circle Diagram Approach for Global Analysis of Transmission System Performance, *IEEE Trans on Power Apparatus and Systems*, Vol. 103, No. 10, **October 1984**.

Hybrid dynamics

- Analytical investigations, such as bifurcation analysis, are vital for establishing fundamental behavioural properties of power systems.
  - Even more important for emerging power systems with large-scale converter-based sources.
- Discrete/switching events usually contribute to disturbances.
- It is **not sufficient** to say power system dynamics can be described by a differential-algebraic model:

$$\dot{x} = f(x, y), \qquad x(0) = x_0$$
$$0 = g(x, y)$$

- Discrete/switching events must be incorporated systematically.

Systematic event modelling

- In response to a **triggering** event, there may be many **consequent** events.
- Example: Protection on a transmission line encounters a trip condition (triggering event), the transmission line disconnects, network voltages undergo a step change, some converter-based resources alter operational modes (consequent events).
- This setting can be captured through a switched differential-algebraic model,

$$\dot{x} = f(x, y), \quad x(0) = x_0$$
$$0 = g_e(x, y)$$

- where g_e is indexed by events $e \equiv [e_1, ..., e_{n_e}]$ with each $e_i \in \{-1, 1\}$ providing the event status.
- Each event e_i corresponds to an algebraic variable y_j such that $e_i y_j \geq 0$, where $e_i y_j = 0$ implies $y_j = 0$, which is the **triggering** condition for event e_i.

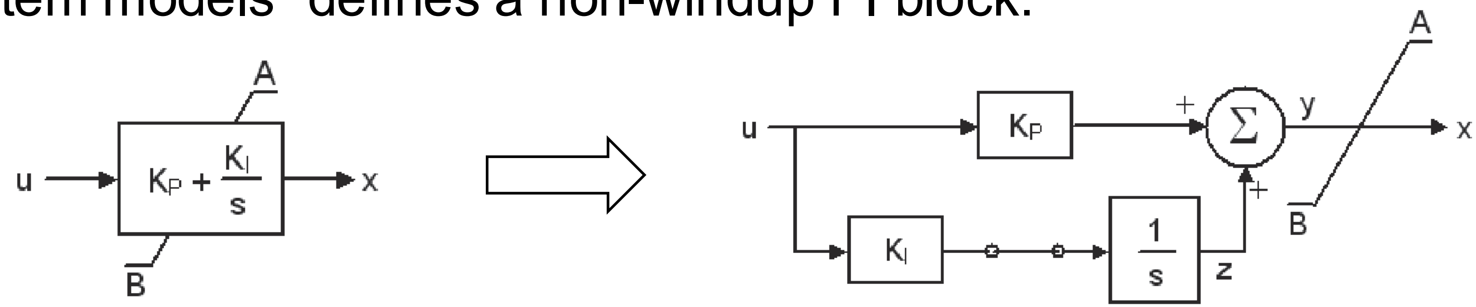
Event modelling (continued)

- As algebraic variables adjust to satisfy $g_e(x, y) = 0$ some status variables e_i may switch sign to maintain $e_i y_j \geq 0$.
- These are **consequent** events.
  - They occur at the same time instant as the triggering event, but only occur because of the triggering event.

- This framework is amenable to automated model building.
- It also provides a structure for checking model integrity, for example preventing switching deadlock (chattering Zeno).

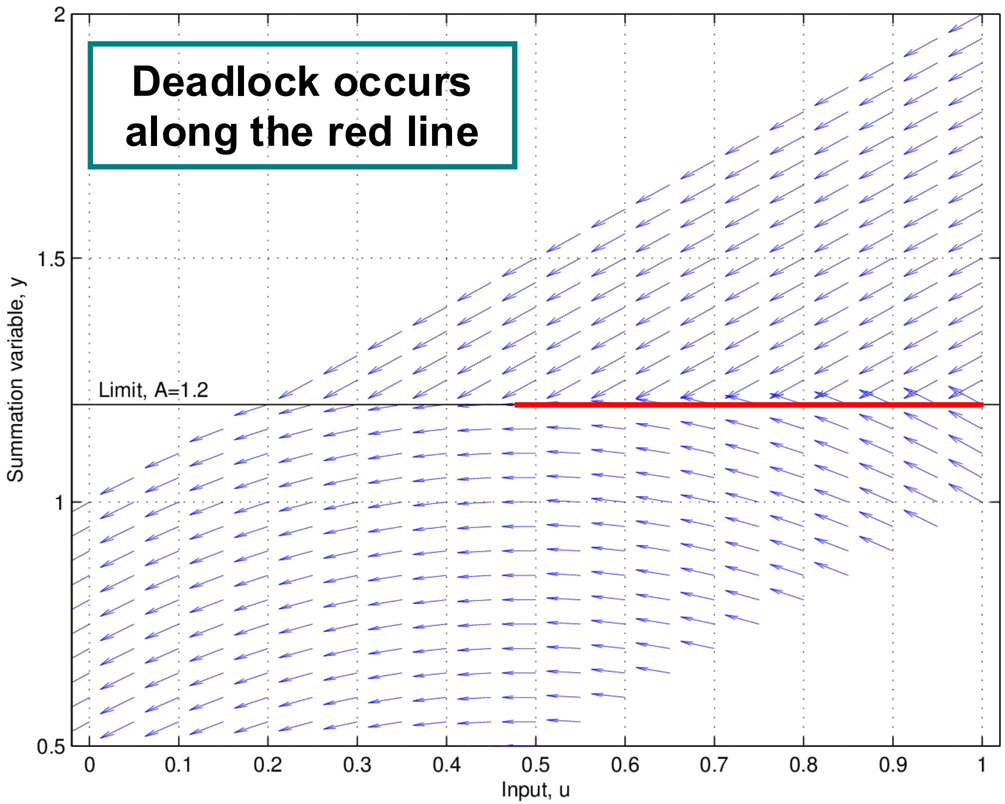
Deadlock

- IEEE Standard 421.5-2016 “Recommended practice for excitation system models” defines a non-windup PI block.



$$\text{If } y > A, \quad \text{then } x = A \text{ and } \frac{dz}{dt} = 0$$

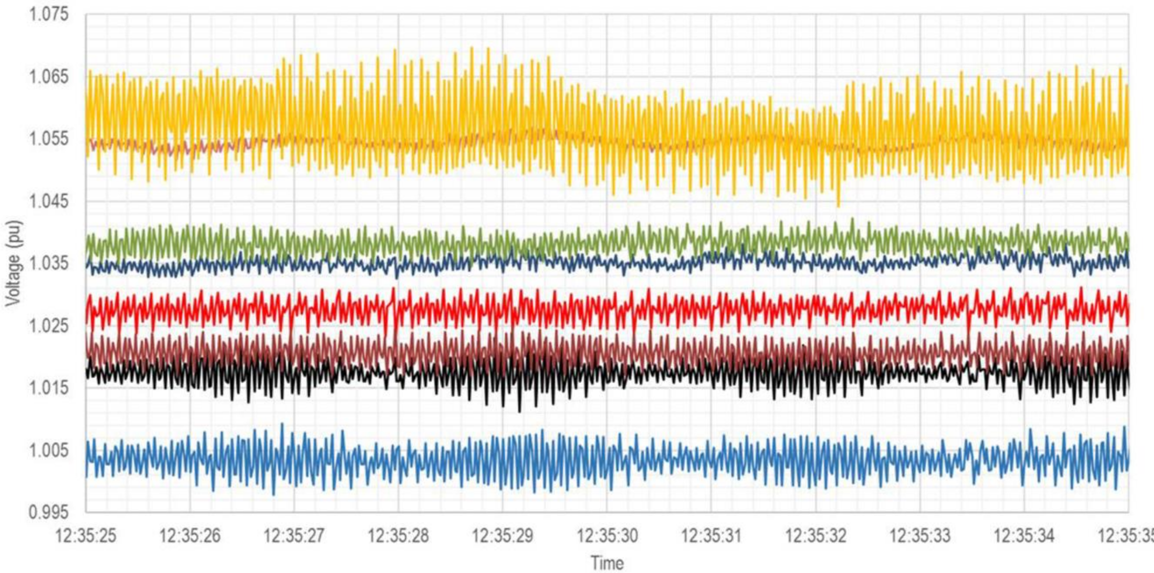
$$\text{If } y < B, \quad \text{then } x = B \text{ and } \frac{dz}{dt} = 0$$



- Response by Standards Committee: “It doesn’t show up in industry studies so what’s the problem?”
- Incorrect modelling together with incorrect numerical integration gives useable results, apparently!
- Other models use this same logic.

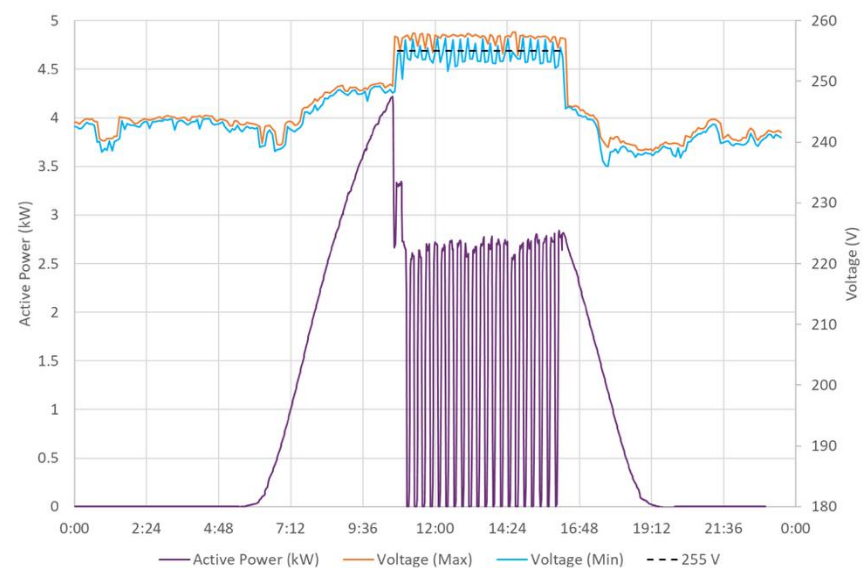
Hiskens, “Trajectory deadlock in power system models”, ISCAS 2011.

Nonlinear oscillations



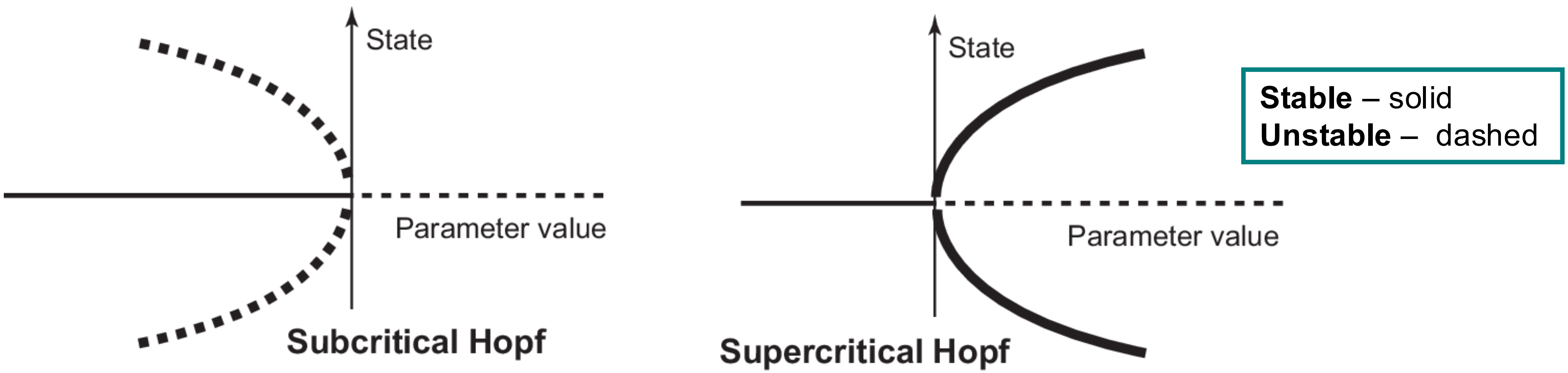
West Murray Zone Power System Oscillations 2020-2021, AEMO, Feb 2023.

J. Riesz, "Operating a power system on 100% distributed resources", AEMO presentation.



Linear systems

- Linear analysis can reveal transitions between stable and unstable oscillations. (Hopf bifurcations.)



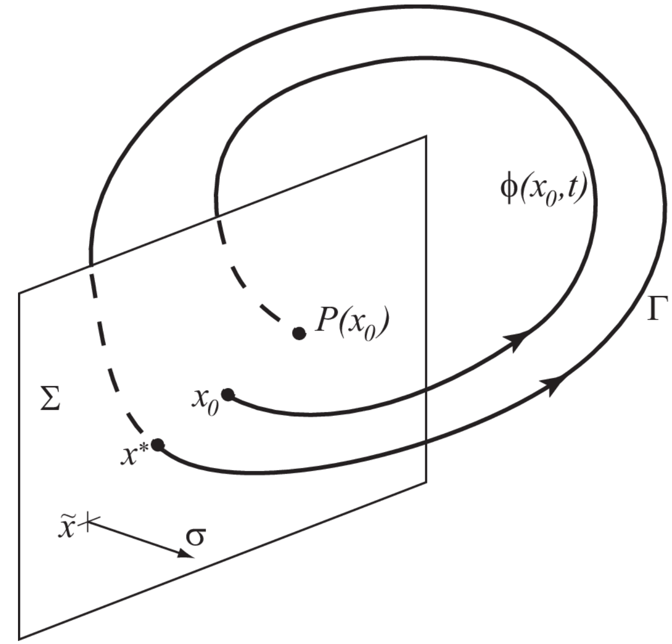
- Linear systems cannot exhibit sustained oscillations (except at the exact bifurcation point).

- So why is there such an obsession with linear analysis for assessing power system oscillations?

Limit cycles (sustained oscillations)

- Solve $F_l(x) := \phi(x, \tau_r(x)) - x = 0$

  where $\phi(x, \tau_r(x))$ denotes the trajectory starting at x and running for time $\tau_r(x)$, called the return time.



- Shooting method solution uses Newton iteration, $x^{k+1} = x^k - \left(DF_l(x^k)\right)^{-1} F_l(x^k)$ where

$$DF_l(x^k) = \left(I - \frac{f|_{\tau_r(x^k)} \sigma^{\mathsf{T}}}{\sigma^{\mathsf{T}} f|_{\tau_r(x^k)}} \right) \Phi(x^k, \tau_r(x^k)) - I$$

- Trajectory sensitivities $\frac{\partial x}{\partial x_0}(t)$ are denoted by $\Phi(x, \tau_r(x))$.
- The vector σ is normal to the Poincare surface Σ .

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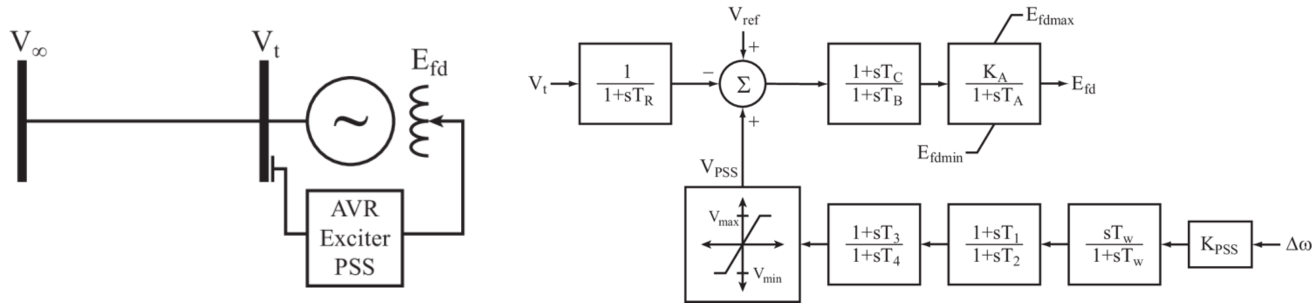
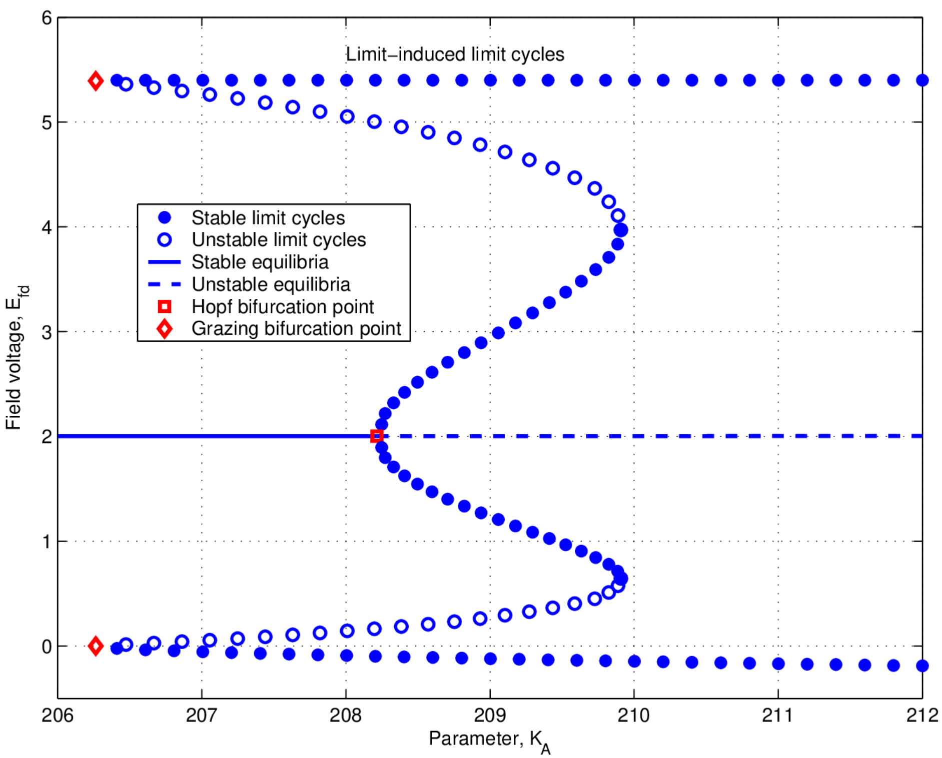
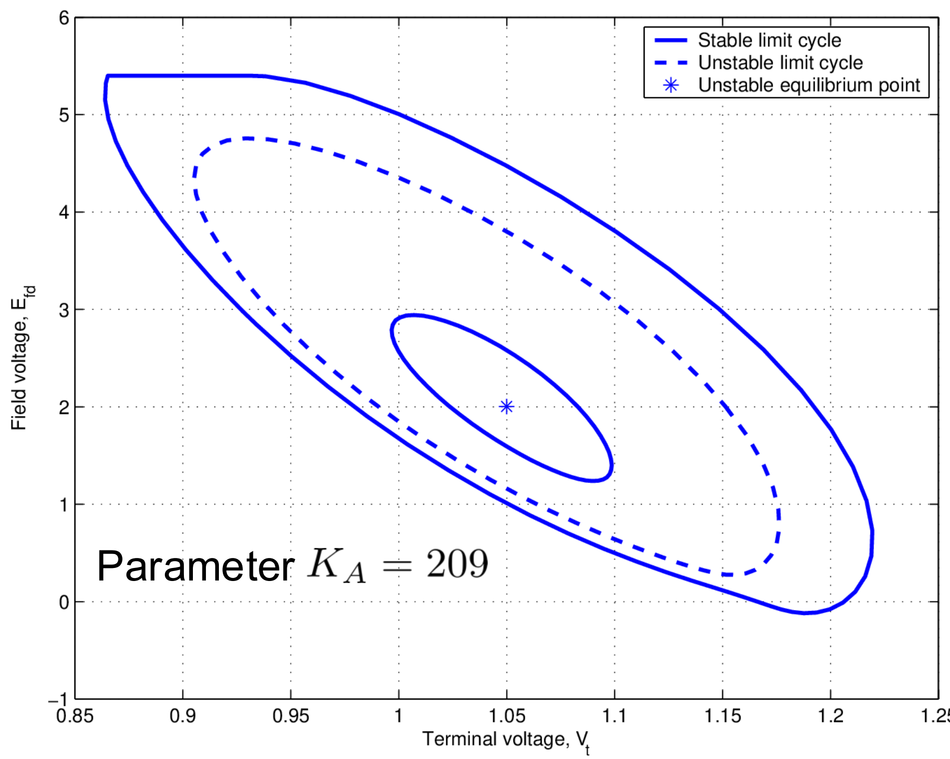
Limit cycle example

Single machine infinite bus with AVR, exciter and PSS







Hiskens and Reddy, "Switching-induce stable limit cycles", *Nonlinear Dynamics*, 2007.

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Conclusions

- Satisfying system strength requirements is enormously expensive.
- What is actually meant by system strength?
  - Need to think carefully about the engineering behind the concept.
  - SCR-related concepts are probably no longer appropriate.
  - Need agreement across organizations.
- Why are utilities asked to meet stringent system strength requirements when equipment manufacturers could use better controller designs?
- Power systems are hybrid dynamical systems.
  - Accurate handling of discrete/switching events is vital to have any chance of replicating power system behaviour.
- Systematic modelling of power systems provides a basis for more incisive simulation-based analysis through formulation of inverse (boundary value) problems and solution using shooting methods.


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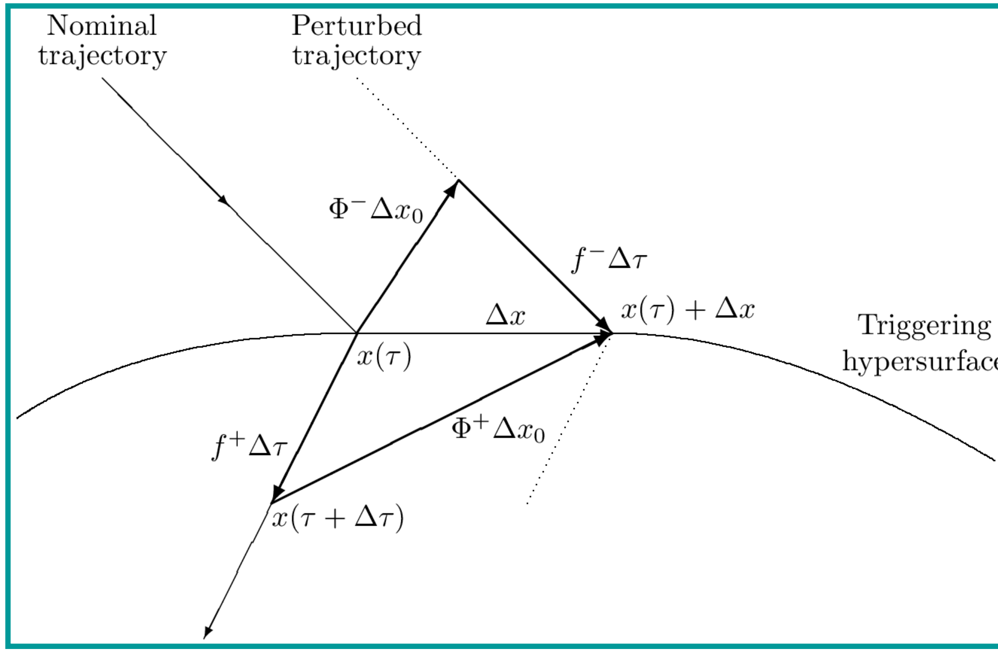
Answers…   42

Mapping sensitivities through events

- Events have two fundamental attributes, the trigger and the response.
- Consider the simplified model, $\dot{x} = f(x), \quad x(0) = x_0$.
- Event triggering occurs when $s(x) = 0$.
- Trajectory sensitivities highlight the role of the two event attributes:

$$\Phi^+ = \Phi^- - (f^+ - f^-)\frac{\partial \tau}{\partial x_0}$$

where $$\frac{\partial \tau}{\partial x_0} = -\frac{(\frac{\partial s}{\partial x})^- \Phi^-}{(\frac{\partial s}{\partial x})^- f^-}$$



- Note the influence of the trigger through $\frac{\partial s}{\partial x}$ and the response $(f^+ - f^-)$.

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