

Role of Rheology in Extrusion – Determining the Effective Viscosity in the Screw

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Effective Viscosity in the Screw Influence of Rheology on Extrusion

The Rheology of an Extrusion Melt considers both the Viscous Effects and also the Elastic Effects associated with the Melt Flow.

The Rheology has a direct influence on the Product Characteristics via:

- Control of the Bubble Growth Rate
- Control of the Degree of Expansion
- Control of the Degree of Curvature



The Rheology has a direct influence on the Process Design via:

- Control of the Die Pressure Drop
- Control of the Screw Flow Dynamics
- Control of the Screw Energy Consumption



Effective Viscosity in the Screw

Extrusion System Volumetric Flow Components

$$Q_{\text{net}} = Q_{\text{drag}} - Q_{\text{pressure}} - Q_{\text{leakage}} = Q_{\text{net}} = K \Delta P / \mu_{\text{apparent}}$$

Screw Characteristic

Die Characteristic

$$Q_{\text{net,in}} \approx M / \rho_B$$

$$Q_{\text{net,out}} = Q_{\text{net,in}} - Q_{\text{gas}} = M / \rho_M$$

where,

$$Q_{\text{net}} = \text{Total Melt Flow} / \text{Melt Density} \quad [\rho_M = f[\text{Recipe}, w_M, T_M]]$$

$$Q_{\text{drag}} = f[N, \text{Screw Geometry}, \text{Recipe}, \text{COF}_{\text{barrel}}]$$

$$Q_{\text{pressure}} = f[\Delta P, H^3, 1 / \mu_{\text{apparent}}]$$

$$Q_{\text{leakage}} = f[\Delta P, \text{Flight Gap}, 1 / \mu_{\text{apparent}}]$$

$$K = f[A, L, S, \text{COF}_{\text{die}}]$$

Note: Sufficient Recirculation must occur or the Product won't be Cooked.

Effective Viscosity in the Screw

How can we define the Viscosity in the Screw?

If we could look inside an Operating Extruder (in Steady-State) we would find a Substrate (i.e. the Formulation plus all the Additives) which is in a continual state of "flux" or "transformation" (i.e. it's being Cooked!). The question most often asked is:

"Which is the most relevant "Component" of the Rheology?"



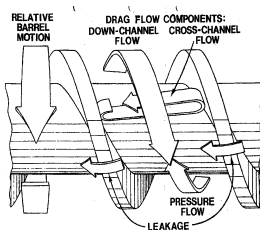
Powder Rheology Dough Rheology Transitional Rheology Melt Rheology Expanded Foam Rheology

The answer is that "the Extruder (i.e., the Motor Drive) "sees" all of them, at the same time!"

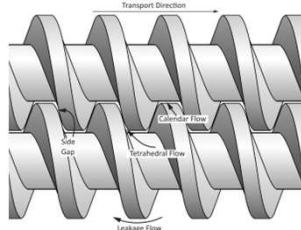
Effective Viscosity in the Screw

Flow Components within the Extruder System

Single Screw Extruders



Co-Rotating TSE



The Flow Components (and hence the Cooking Reactions) involved in the different types of Extruders are also very different.

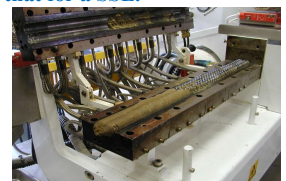
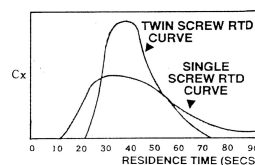
Effective Viscosity in the Screw

The Filled Length of the Extruder

The "balance point" between the various Flow Components, ultimately determines the Filled Length (or the Choke Point) of the Extruder. This determines the Operating Volume and hence, the magnitude of the Mean Residence Time, t_R .

$$t_R = V_{\text{total}} / Q_{\text{net}} = L_f A_x / Q_{\text{net}}$$

The magnitude of t_R for the TSE is typically slightly "lower" (and the RTD is "narrower") than that for a SSE.



Effective Viscosity in the Screw

Energy Inputs

$$E_{\text{total}} = E_{\text{mechanical}} + E_{\text{convective}} + E_{\text{conductive}}$$

Total Energy Input	Net Power Input	Latent Energy Input / Removal	Barrel Heat Transfer
[Typically	40 to 90%]	0 to 30%]	10 to 30%]

where

$$E_{\text{mechanical}} = f[\text{Rheology, Strain}]$$

$$E_{\text{convective}} = f[\text{Volatiles* Injection / Removal}] \quad (\text{Typically Steam})$$

$$E_{\text{conductive}} = f[\text{Barrel Temperature}]$$

Note : The Energy Inputs are **NOT** directly interchangeable! The Energy Inputs determine the *Product Attributes*.

Effective Viscosity in the Screw

Manifestations of the Energy

$$E_{\text{total}} = E_{\text{sensible}} + E_{\text{cook}} + E_{\text{pump}} + E_{\text{screw}} + E_{\text{radiant}}$$

Total Energy Outputs	Temperature Rise	Ingredients Modifications	Pressure Rise	Screw Flow Dynamics	Energy Losses
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[Typically only 1% to 2% of E_{total}]

where,

$$E_{\text{sensible}} = f[\text{Formulation, } T_{\text{ambient}}, T_{\text{preconditioner}}, T_{\text{final}}]$$

$$E_{\text{cook}} = f[\text{Formulation, Cooking Enthalpy}]$$

$$E_{\text{pump}} = f[Q_{\text{net}}, \Delta P]$$

$$E_{\text{radiant}} = f[T_{\text{barrel}} - T_{\text{ambient}}]$$

Note: Achieving these *Process Parameters* is confirmation that we have made the "right" Product.

Effective Viscosity in the Screw

Viscous Dissipation and the RTD

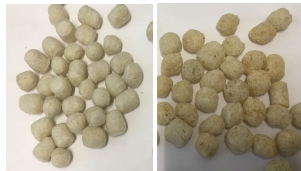
It can be shown (via *Weighted Average Total Strain Analysis*) that the *Power Dissipation* within the Extruder is a function of the *Rheology* of the *Formulation*, the *Residence Time* and the *Shear Rate* developed within the Extruder Screw.

As a result of the complex *Flow Dynamics* developed within the various types of Extrusion systems, the *Product Melt* will experience a *Residence Time Distribution (RTD)* rather than a single value of *Residence Time*.

The significantly different *Extruder Geometry* in the various parts of the Extruder also results in different *Shear Rates* (both in terms of magnitude and direction).

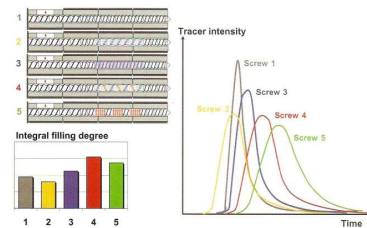
This can result in very "different" looking Products being prepared from alternative Extruders, even with the same *Ingredients*.

Two very different looking Products prepared using the same Extruder and the same *Formulation*.



Effective Viscosity in the Screw

The *Residence Time Distribution (or RTD)*, however, is determined by the specific *Screw Geometry* and the *Screw Speed*.



Each different *Screw Design* results in a different *Degree of Fill (or Filled Length, L_f)* and hence a different *Average Residence Time, t_{ave}* and a different *RTD* and different *Power Dissipation*.



$$\begin{aligned} \text{Average Residence Time} &= \theta_{\text{av}} = [\theta_{\text{min}} + \theta_{\text{max}}] / 2 \\ \text{Standard Deviation} &= \sigma = [\theta_{\text{max}} - \theta_{\text{min}}] / 4 \\ \text{Coefficient of Variance} &= \text{COV} = \sigma / \theta_{\text{av}} \end{aligned}$$

Effective Viscosity in the Screw

Shear Rates in the different Parts of the Screw

In a given Extruder System the most important *Shear Rates* are:

The *Flight Gap Shear Rate*, The (Minimum) *Channel Shear Rate*,

$$\gamma_{\text{gap}} \approx \pi D_s N / g = \Omega_g N$$

[results in "Cooking"]

$$\gamma_{\text{channel,min}} \approx \pi D_s N / H$$

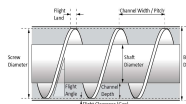
[results in "Mixing"]

The (Average) *Channel Shear Rate*,

$$\gamma_{\text{channel,ave}} \approx \pi N (R_s + S) / (R_s - S) = \Omega_c N$$

The *Apparent Viscosity* in the various parts of the Screw will be proportional to

$$\mu_g / \mu_c = [\Omega_c / \Omega_g]^{(1-n)}$$



Typical Single Screw Geometry

Effective Viscosity in the Screw

Estimation of the Viscous Power Dissipation - SSE

For a Single Screw Extruder (SSE), the *Viscous Power Dissipation, P_{net}* , within the Extruder Screw may be estimated via the following simple expression

$$P_{\text{net}} \approx \pi^3 N^2 D^3 \{ [\pi D \mu_c / (2 H)] + [\#_f e \mu_g / g] \} L_f$$

where,

P_{net} = Power (W), N = Screw Speed (Hz), D = Screw Diameter (m), H = Channel Depth (m), $\#_f$ = Flight Starts (-), g = Flight Gap (m), e = Flight Thickness (m), μ_c = Channel Melt Viscosity (Ns m⁻²), μ_g = Flight Gap Melt Viscosity, (Ns m⁻²) and L_f = Screw Filled Length (m)

For a Screw of known *Geometry* this may be simplified to

$$P_{\text{net}} \approx \Psi_1 \{ [\Psi_2 \mu_c] + [\Psi_3 \mu_g] \}$$

where,

$$\Psi_1 = [\pi^3 N^2 D^3 L_f], \Psi_2 = [\pi D / (2 H)] \text{ and } \Psi_3 = [\#_f e / g]$$

A Process-dependent Term

Purely Geometric Terms

Effective Viscosity in the Screw

Estimation of the Viscous Power Dissipation - SSE

Based upon the expression presented above, it is clear that if we know the *Screw Geometry*, the *Screw Speed*, the *Product Rheology* and the *Filled Length*, it is possible to determine the *Viscous Power Dissipation*.

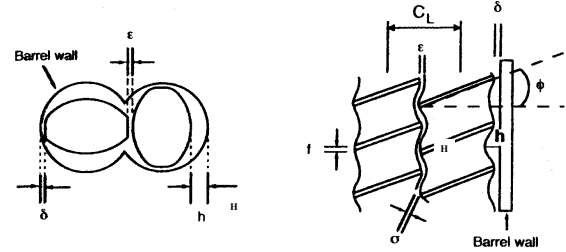
Conversely, if we know the *Screw Geometry* and the *Screw Speed* and then if we measure the *Filled Length* and the *Viscous Power Dissipation*, then it should be possible to determine the *Product Rheology*.

Effective Viscosity in the Screw

Estimation of the Viscous Power Dissipation - TSE

Due to the significantly different *Geometry* for a TSE, the estimation of the Power requires an alternative approach.

The relevant Geometry is shown.



Effective Viscosity in the Screw

Estimation of the Viscous Power Dissipation – TSE

Determination of the Volume and the Shear Rate TSE

Zone	Volume	Shear Rate
Screw Channel	$V_{\text{channel}} = n \pi^2 D_e D_s H \tan \phi / 2$	$\gamma_{\text{channel}} = \pi D_e N / H$
Flight Gap	$V_{\text{gap}} = n f C_e \delta$	$\gamma_{\text{gap}} = \pi D_e N / \delta$
Tip to Channel Bottom	$V_{\text{tip}} = 2 n f C_L \varepsilon$	$\gamma_{\text{tip}} = 2 \pi C_L N / \varepsilon$
Flight Flanks	$V_{\text{flanks}} = n [(D_s^2 - C_L^2)]^{0.5} H \sigma / 2$	$\gamma_{\text{flanks}} = \pi C_L N / \sigma$

where,

$$\text{Equivalent Screw Diameter, } D_e = [2 D_s] - [0.9003 (D_s H)^{0.5}]$$

$$\text{Circumference of Equivalent Screw, } C_e = 2 [(\pi D_s) - (2 D_s H)^{0.5}] = \pi D_e$$

Effective Viscosity in the Screw

Estimation of the Viscous Power Dissipation - TSE

(1) The Power Dissipated in the *Screw Channels*, Z_{channel} , is given by

$$Z_{\text{channel}} = [\mu_{\text{ave}} \#_F N^2] [n \pi^4 D_s^3 D_e \tan \phi / 2 H]$$

(2) The Power Dissipated in the *Flight Gap*, Z_{gap} , is given by

$$Z_{\text{gap}} = [\mu_{\text{ave}} \#_F N^2] [n \pi^2 D_e^2 f C_e / \delta]$$

(3) The Power Dissipated in the *Calendar Gap*, Z_{calendar} , is given by

$$Z_{\text{calendar}} = [\mu_{\text{ave}} \#_F N^2] [8 n \pi^2 C_L^3 f / \varepsilon]$$

(4) The Power Dissipated in the *Screw Flanks*, Z_{flanks} , is given by

$$Z_{\text{flanks}} = [\mu_{\text{ave}} \#_F N^2] [n \pi^2 C_L^2 H \{D_s^2 - C_L^2\} / (2 \sigma)]$$

Similar (but more complex) arguments, as above for the SSE, can be made for the TSE, which shows that the Power is simply a function of *Screw Speed*, *Geometry* and *Rheology*.

Effective Viscosity in the Screw

Equivalence of Electrical and Mechanical Energy

Many Food Processing Operations involve the use of *Mechanical / Electrical methods to input Energy* e.g. *Rotating Shafts or Compression Rolls*. The *Mechanical Energy Input* are normally described via either *Motor Load* or *Motor Current* or *Torque Loading*.

Due to the *Inefficiencies* involved with the various alternative *System Designs*, the *Energy Consumption under No Load* must also be allowed for.

The *Specific Mechanical Energy* (SME) is the parameter typically used to report the *Mechanical Energy Input*.

Effective Viscosity in the Screw

Equivalence of Electrical and Mechanical Energy

The *Net Energy Input*, P_{net} (kW), can be estimated via

$$E_{\text{mechanical}} = \sqrt{3} V [(\phi \eta I) - (\phi_0 \eta_0 I_0)] / 1000 \quad \text{for AC Power}$$

where,

$$\phi = \text{Power Factor (-)} \quad \eta = \text{Efficiency (\%)}$$

$$V = \text{Supply Voltage (V)} \quad I = \text{Current (Amps)}$$

$$\text{Subscript 0 = No Load}$$

Motor Current Meter



$$E_{\text{mechanical}} = 2 \pi (\tau - \tau_0) N / 60000$$

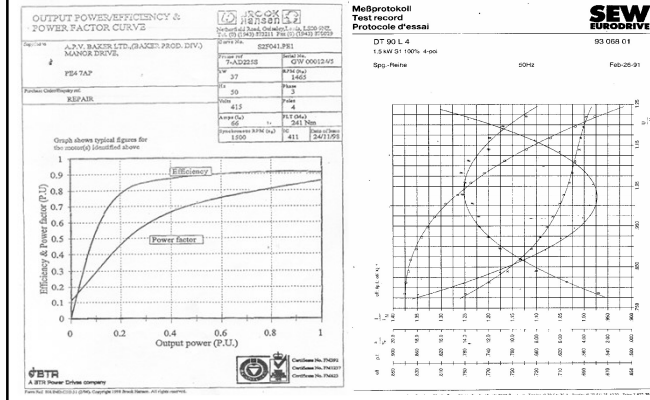
for DC Power

$$E_{\text{mechanical}} = V [(\eta I) - (\eta_0 I_0)] / 1000$$

where,

$$\tau = \text{Torque (N m)} \quad N = \text{Shaft Speed (rpm)}$$

Effective Viscosity in the Screw AC Motor Performance Curves



Effective Viscosity in the Screw

Typical Performance Data for DC Motors

Power		Efficiency			
(hp)	(kW)	0.25	0.5	0.75	1
0.25	0.186	58.0	62.0	65.0	68.0
0.50	0.373	61.0	65.0	70.0	72.0
1.00	0.746	66.0	70.0	73.0	75.0
1.50	1.119	67.0	72.0	76.0	78.0
2.00	1.491	68.0	72.0	77.5	81.0
3.00	2.237	69.0	72.0	77.0	79.0
5.00	3.728	71.0	77.0	81.0	81.0
7.50	5.593	73.0	79.0	82.0	83.0
10.00	7.457	76.0	81.0	83.0	85.0
15.00	11.185	77.0	81.5	84.5	86.0

For Baldor Electric Motors

Effective Viscosity in the Screw Calculating the SME

The calculation of the SME requires that the *Total Mass Flow* (M_{Total}) be determined via

$$M_{\text{Total}} = M_{\text{Dry Powder}} + M_{\text{Water}} + M_{\text{Steam}} + M_{\text{Oil}} + M_{\text{Additives}}$$

where,

$M_{\text{Dry Powder}}$ = Mass Flow of Dry Powder (kg / hr)

M_{Water} = Mass Flow of Water (kg / hr)

M_{Steam} = Mass Flow of Steam (kg / hr)

M_{Oil} = Mass Flow of Oil (kg / hr)

$M_{\text{Additives}}$ = Mass Flow of other Additives (kg / hr)

The SME is then given by

$$SME_1 = P_{\text{net}} / M_{\text{Total}} \quad (\text{kW hr / kg})$$

or

$$SME_2 = 3600 SME_1 \quad (\text{kJ / kg})$$

Effective Viscosity in the Screw

Experimental Methodology

In order to determine the *Effective Viscosity*, μ_{eff} , of the Material within the Extruder Screw, follow these steps:

Step 1 - Operate the Extruder (of known Geometry) in Steady-State, at a given Processing Rate, M_1 , with a known Recipe [D , W , w_M , T_M , etc.], a given Screw Speed, N_A .

Step 2 - Determine the Net Power, $P_{\text{net},1}$ and the Specific Mechanical Energy, SME_1 .

Step 3 - Measure the Average Residence Time, $t_{\text{ave},1}$ and from this determine the Filled Length, L_{f1} .

Step 4 - Increase the Processing Rate (to $M_2 \approx 1.2M_1$), whilst keeping the Recipe and Screw Speed the same.

Step 5 - Repeat Steps 2 and 3 to obtain $P_{\text{net},2}$, SME_2 , $t_{\text{ave},2}$ and L_{f2} .

Effective Viscosity in the Screw

Experimental Methodology, cont.,

Step 6 - Solve the Power Equation for the two unknowns $\mu_{c,A}$ and $\mu_{g,A}$ (at Screw Speed, N_A).

Step 7 - Increase the Screw Speed (to $N_B \approx 1.2 N_A$), then repeat Steps 2 to 6.

Step 8 - Increase the Screw Speed (to $N_C \approx 1.2 N_B$), then repeat Steps 2 to 6.

Step 9 - By repeating this procedure over a range of Screw Speeds, it will be possible to gather sufficient Data to make a Plot of $\ln[\mu_{\text{eff}}]$ vs. $\ln[N]$, which will be a Straight Line. The Slope of the Line will be equal to the value $(n - 1)$, where n is the Flow Behaviour Index.

Step 10 - Determine the magnitude of the Consistency Index, m .

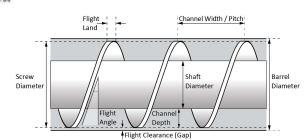
Note: The Product Quality generated (i.e., the Shape, Size, Density, etc.) is of no consequence during this study.

Effective Viscosity in the Screw

A Worked Example – Extruder and Die Geometry

Extruder Data

Screw Diameter, D_s	130.0	mm
Total Screw Length, L_s	2000.0	mm
Flight Gap, g	1.5	mm
Flight Width, e	12.0	mm
Number of Flight Starts, $\#_f$	2	starts
Inlet Channel Depth, B_{in}	35.0	mm
Inlet Length, L_i	900.0	mm
Transition Length, L_t	580.0	mm
Metering Channel Depth, B_m	25.0	mm
Inlet Length, L_i	600.0	mm
Channel Width, W	50.0	mm
Flight Angle, α	12.0	deg
Flow Area for Product, A_p	9.235E-03	m ²
Geometric Constant 2, ψ_2	7.0013	-
Geometric Constant 3, ψ_3	16.0000	-

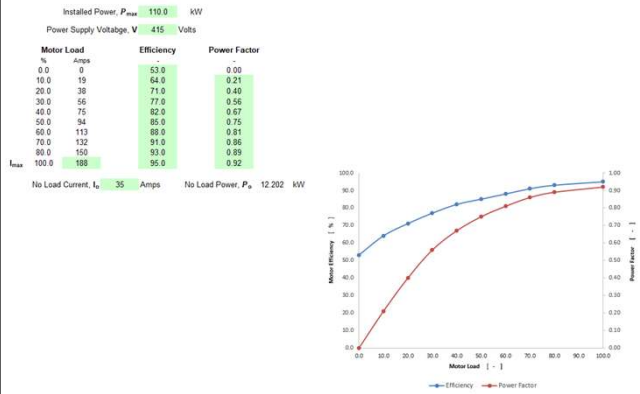


Die Data

Die Radius, R_d	1.50	mm
Land Length, L_d	6.00	mm
Number of Dies, $\#_d$	120	holes
Die Hole Conductance, k_h	0.3313	mm ³ per Hole
Total Conductance, K_T	39.761	mm ³
Die Open Area, A_d	3.802E-04	m ²
Die Assembly Volume, V_d	2.590E-04	m ³

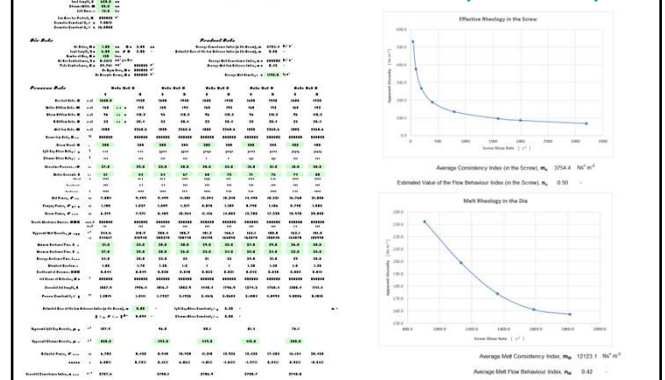
Effective Viscosity in the Screw

A Worked Example – Motor Performance Data



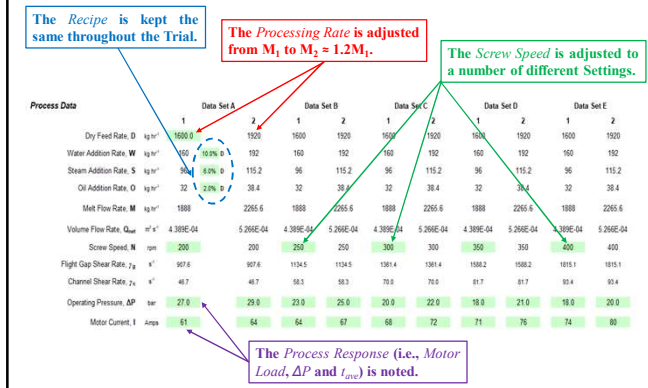
Effective Viscosity in the Screw

A Worked Example – Trial Summary & Data Analysis



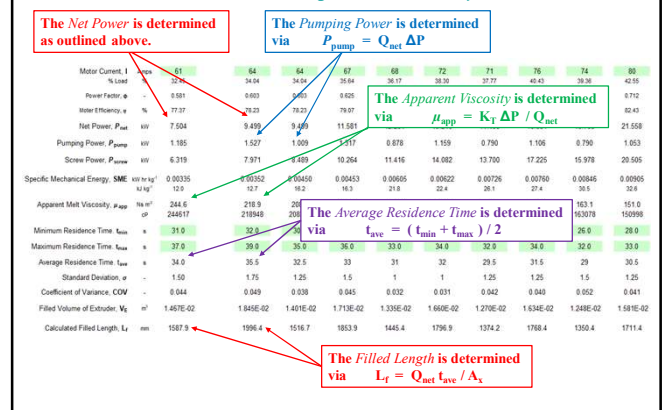
Effective Viscosity in the Screw

A Worked Example – Process Data



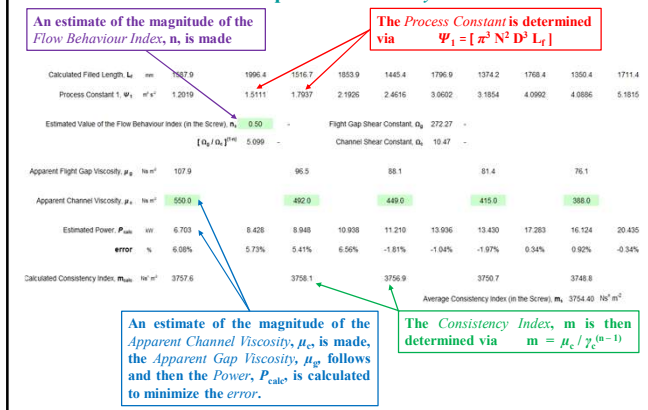
Effective Viscosity in the Screw

A Worked Example – Data Analysis



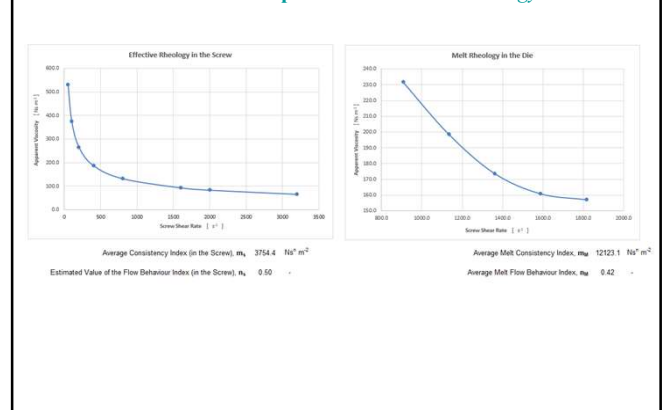
Effective Viscosity in the Screw

A Worked Example – Data Analysis cont.



Effective Viscosity in the Screw

A Worked Example – Formulation Rheology



Effective Viscosity in the Screw

Summary

The numerous *Physical & Chemical Transformations* of the *Formulation*, which occur within the **Extruder Screw**, will ultimately result in the formation of the *Melt*.

These changes are solely dependent upon the interaction of the *Screw Geometry*, the *Rotational Speed* and the *Effective Formulation Rheology* (which is continuously changing along the *Length of the Screw*) thereby determining the *Strain (or Work Input)* for the Process.

A (“simple”) *Methodology for estimating the Effective Viscosity (or the Global Rheological Properties)* has been presented herein.

An Understanding (and the Control) of the Rheological Properties of the Formulation Ingredients (i.e., the Ingredient Quality) is therefore of paramount importance to ensure optimal Process Control for any Extrusion Operation.